1) Find the derivatives of the following functions. State the rule you are using.
(a) \( f(x) = 4x^3 - \frac{5}{x^3} \)

Here we use the power and sum rules.
\[
\frac{df}{dx} = \frac{df}{dx}(4x^3 - 5x^{-3})
= 4 \cdot 3x^{3-1} - 5 \cdot (-3)x^{-3-1}
= 12x^2 + \frac{15}{x^4}.
\] 
(b) \( g(z) = z^2 \sin(z) \)

Here we use the product and power rules.
\[
\frac{dg}{dz} = \frac{dz^2}{dz} \sin(z) + z^2 \frac{d\sin(z)}{dz}
= 2z \sin(z) + z^2 \cos(z).
\] 
(c) \( h(x) = \frac{\sin(x)}{x^2 + 4} \)

Here we use the quotient and power rules.
\[
\frac{dh}{dx} = \frac{\frac{d\sin(x)}{dx}(x^2 + 4) - \sin(x) \frac{d(x^2 + 4)}{dx}}{(x^2 + 4)^2}
= \frac{\cos(x)(x^2 + 4) - 2x \sin(x)}{(x^2 + 4)^2}
\] 
(d) \( s(w) = \sin\left((\cos(w) + w^2)^2\right) \)

Here we use the chain rule three times and the power rule.
\[
\frac{ds}{dw} = \cos\left((\cos(w) + w^2)^2\right) \frac{d}{dw} (\cos(w) + w^2)^2
= \cos\left((\cos(w) + w^2)^2\right) 2 (\cos(w) + w^2) \frac{d}{dw} (\cos(w) + w^2)
= \cos\left((\cos(w) + w^2)^2\right) 2 (\cos(w) + w^2) (-\sin(w) + 2w)
\]
(e) \( \vec{r}(t) = (t^2 + t, \cos(2t), \sin(3t)) \)

Here we use that the components of the derivative of a vector valued function are the
derivatives of the components, the power rule and the chain rule.

\[
\frac{d\vec{r}}{dt} = \left( \frac{d(t^2 + t)}{dt}, \frac{d\cos(2t)}{dt}, \frac{d\sin(3t)}{dt} \right) = (2t + 1, -2\sin(2t), 3\cos(3t))
\]

2) If \( x^3y - x - y + y^4x^2 = 0 \) defines \( x \) as a function of \( y \) at \((1, 1)\), find \( x'(y) \) at the point \((1, 1)\).

Rewriting to be explicit gives

\[
x(y)^3y - x(y) - y + y^4x(y)^2 = 0.
\]

Differentiating with respect to \( y \) gives

\[
3x^2x'y + x^3 - x' - 1 + 4y^3x^2 + 2y^4xx' = 0.
\]

At this point one can set \( x = 1 \) and \( y = 1 \) to get

\[
3x' + 1 - x' - 1 + 4 + 2x' = 0
\]

This reduces to \( 4x' + 4 = 0 \) or \( x' = -1 \).

3) Consider the function \( z(x) = \frac{x^2 - 4x + 3}{2x^2 + 4x + 2} \).

If I know I did not leave enough space to work this problem. I am sorry.

(a) Find the critical points of \( z(x) \) and all asymptotes of \( z(x) \).

First note that the denominator is 0 at \( x = -1 \) and the numerator is not zero. This
means that there is a vertical asymptote at \( x = -1 \).

For horizontal asymptotes we may assume that \( x \neq 0 \) to rewrite the function as

\[
z(x) = \frac{1 - \frac{4}{x} + \frac{3}{x^2}}{2 \left(1 + \frac{2}{x} + \frac{1}{x^2}\right)}.
\]

It is fairly easy to take the limits at \( \infty \) and \( -\infty \) to get

\[
\lim_{x \to \infty} z(x) = \lim_{x \to -\infty} z(x) = \frac{1}{2}.
\]

This gives a horizontal asymptote at \( y = 1/2 \).

To find the critical points on sets \( z'(x) = 0 \) and solves for \( x \). Skipping a few steps one
gets

\[
z'(x) = \frac{3x - 5}{(x + 1)^3}.
\]

This is 0 at \( x = 5/3 \). The value \( x = -1 \) is not a critical point since it is not in the
domain of \( z(x) \).
(b) Where is $z(x)$ increasing? Where is it decreasing?

One only needs to find where the derivative is positive and where it is negative. Since the points where the sign of $z'(x)$ can change are $-1$ and $5/3$ one only needs to check the values of $z'(x)$ at $-2$, $0$, and $2$. These values are $11$, $-5$, and $1/27$. Therefore $z$ is increasing on $(-\infty, -1) \cup (5/3, \infty)$ and $z$ is decreasing on $(-1, 5/3)$.

(c) Where is $z(x)$ concave up? Where is it concave down?

Here we need to find the sign of the second derivative over intervals. Calculating, which I am skipping, gives

$$z''(x) = \frac{6(3 - x)}{(x + 1)^4}.$$  

The only places this can change sign are at $x = -1$ and $x = 3$. Evaluating at appropriate points give that $z$ is concave up on $(-\infty, -1)$ and $(-1, 3)$. The function $z(x)$ is concave down on $(3, \infty)$.

(d) Sketch the graph of $z(x)$ noting the features you discovered in the previous parts of this problem.

Here is the plot using a computer algebra system.
4) A 20 foot flag pole casts a shadow 30 feet long. An ant is climbing up the flag pole at a rate of 1 foot per minute. The ant also casts a shadow. How fast is shadow of the top of the ant’s head moving when the ant’s head is 10 feet off the ground?

By similar triangles one has 
\[
\frac{L}{H} = \frac{30}{20} \quad \text{or} \quad L = \frac{3H}{2}.
\]

Differentiating the second equation with respect to \( t \) gives
\[
\frac{dL}{dt} = \frac{3}{2} \frac{dH}{dt}.
\]

Substituting in \( \frac{dH}{dt} = 1 \) gives \( \frac{dL}{dt} = 1.5 \text{ ft/min} \).

5) Explain, and in words justify if you can, what the chain rule says about the rate of change of a function.

The chain rule simply states, reading the symbols, that the rate of change of the derivative of a composition is the product of the rates of changes of the two functions.